A Mathematical Analysis

of

Boston 5

Prepared for

New Vision Gaming

by

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Rules of Play

Boston 5 is a five-card variation of Boston Stud Poker. It is dealt from a single deck of cards and can be played by six players.

<u>Ante and Wagers</u>

To start the game, each player must Ante and place a first wager. The Ante amount will be half of the first wager. Then each player and the dealer are dealt three cards face down. After viewing their cards, the players have the option to fold or place a second wager equal to their first wager. If they fold, they will forfeit their Ante and first wager. Those players who did not fold their hand will now receive two final cards dealt face down. The dealer also deals himself two final cards.

The player's five-card hand is then compared with the dealer's. Winning hands are paid even money on the wagers and the Ante is returned. If the hand loses, the dealer sweeps both the Ante and the wagers. Ties are pushes. Irrespective of the dealer hand, a Bonus is always paid based on the Ante if the player hand is two pair or better:

Bonus Pay Tables

Hand Type	Table 1	Table 2	Table 3	Table 4	Table 5
Royal Flush	1000	1000	1000	1000	1000
Straight Flush	200	200	200	200	200
4 of a Kind	100	100	100	100	100
Full House	25	25	25	25	25
Flush	15	15	15	15	15
Straight	10	10	10	8	10
3 of a kind	5	4	5	5	4
Two Pair	3	3	2	2	2

<u>3-Card Hand Side Bet</u>

In addition to the Ante and wager, the player has the option to make a side bet on his first three cards. The player is betting against one of the following pay tables:

3-card Bet Pay Tables

Hand Type	Table 6	Table 7	Table 8	Table 9
Straight Flush	40	40	40	40
3 of a Kind	30	30	30	25
Straight	6	5	6	6
Flush	4	4	3	4
One Pair	1	1	1	1
Others	-1	-1	-1	-1

All payoff odds are "to 1."

Mathematical Analysis

After placing an Ante and the first wager the player must make a decision: fold or continue. He will weigh the expected return of his action and will stay in the game if the following criterion is met:

$$4 * W + B - 5 * L \geq -3, \text{ where}$$

> W is the probability of winning at the final showdown,
> L is the probability of losing at the final showdown,
> B is the expected bonus paid on the Ante, and
> -3 is the player's loss if he folds.

Solving for W, we get $4W = 5L - B - 3$.
Assuming that B is small enough to be ignored[‡] and that $L = 1 - W$[§], we get
$$4W = 5 - 5W - 3,$$
$$9W = 2,$$
$$W \approx 22.2\%.$$

In other words, any player 3-card hand that can win at least 22% of the time at the showdown should not be folded. If the worst 3-card hand that can be dealt can win at least 22% of the time then the player should not fold any hand he is dealt. An exact probability analysis indicated that even the worst 3-card hand the player can get, 2-3-7 offsuit, will win 32% of the time:

         Hand analysis of 2♣-3♣-7♥
    Bonus
         3 of a Kind   0.7653% x 5 = 3.8265%
         Two Pair   2.2959% x 3 = 6.8877%
    Total Bonus   10.7143% (x +1)
    Win   32.0972% (x +4)
    Loss   67.8925% (x −5)
    Tie   0.0103%
    Player Expected Value   -2.0035 bets, as opposed to −3 bets if folded

Therefore, the optimal strategy is <u>not to fold at all</u>.

Since the player will play every hand to the end, the player and dealer will split the hands evenly, thereby resulting in a symmetrical probability distribution among them. A computer program was written to determine the probability distribution of the final hands:

[‡] Here we're only interested in the worst hands, which only yield a small bonus award from 3 of a kinds and two pairs.
[§] The probability of a 5-card tie is so small that it can be ignored.

Bonus Pay Table 1

Hand Type	Probability in %	Pays (to 1)	Contribution in %
Royal Flush	0.000154	1000	0.1539
Straight Flush	0.001385	200	0.2770
4 of a Kind	0.024010	100	2.4010
Full House	0.144058	25	3.6014
Flush	0.196540	15	2.9481
Straight	0.392465	10	3.9246
3 of a kind	2.112845	5	10.5642
Two Pair	4.753902	3	14.2617
Total	7.625358		38.1320

The house advantage, for Pay Table 1, can be calculated as follows:

The computer analysis indicated that the player and dealer should beat each other 49.9956% of the time and tie 0.0087% of the time (see Appendix A and its Summary). This causes the player to lose (49.9956% * (5 – 4)) / 5 = 9.9991% per bet unit. The bonus paid on the Ante will reduce the loss to 9.9991% - 38.1320% / 5 = 9.9991% - 7.6264% = **2.3727%** per bet unit.

Hand Type	Table 1	Table 2	Table 3	Table 4	Table 5
Royal Flush	1000	1000	1000	1000	1000
Straight Flush	200	200	200	200	200
4 of a Kind	100	100	100	100	100
Full House	25	25	25	25	25
Flush	15	15	15	15	15
Straight	10	10	10	8	10
3 of a kind	5	4	5	5	4
Two Pair	3	3	2	2	2
House Advantage	2.3727%	2.7953%	3.3235%	3.4805%	3.7461%

<u>3-card Side Bet</u>

Straight Flush = 12 * 4 = 48
3 of a Kind = 13 * C4,3 = 52
Straight = 12 * 4*4*4 – 48 = 720
Flush = C13,3 * 4 – 48 = 286 * 4 – 48 = 1144 – 48 = 1096
One Pair = (13 * C4,2) * 48 = 78 * 48 = 3744

Hand Type	Frequency	Table 6	Table 7	Table 8	Table 9
Straight Flush	48	40	40	40	40
3 of a Kind	52	30	30	30	25
Straight	720	6	5	6	6
Flush	1096	4	4	3	4
One Pair	3744	1	1	1	1
Others	16440	-1	-1	-1	-1
House Advantage		2.3167%	5.5747%	7.2760%	3.4932%

The weighted average house advantage on the 3-card and 5-card bets combined can be computed as follows:

$$A = (B5 * A5 + B3 * A3) / (B5 + B3), \text{ where}$$

$B5$ is the player's total wager on the 5-card bet,
$A5$ is the house advantage on the 5-card bet,
$B3$ is the player's total wager on the 3-card bet, and
$A3$ is the house advantage on the 3-card bet.

Example

The casino offers Pay Tables 3 and 6. Thus, $A5 = 3.3235\%$ and $A3 = 2.3167\%$.
A player places $5 on the 3-card bet and $2.5 on the Ante, $5 on the 1st wager and $5 on the second wager. Thus, $B3 = \$5$ and $B5 = \$12.5$.

Plugging the above numbers into the formula, we get

$$A = (12.5 * 3.3235\% + 5 * 2.3167\%) / (12.5 + 5)$$
$$A = 3.04\% \text{ per total wager}$$

APPENDIX A

Frequency and Results of Player Hand vs. Dealer hand at Showdown (Player never folds)

Player	Dealer	Frequency	Win %	Loss %	Tie %
High Card	High Card	25.280556	49.9843581653	49.9843581653	0.0312836694
High Card	One Pair	21.083614	0.0000000000	100.0000000000	0.0000000000
High Card	Two Pair	2.345956	0.0000000000	100.0000000000	0.0000000000
High Card	3 of a Kind	1.029252	0.0000000000	100.0000000000	0.0000000000
High Card	Straight	0.197921	0.0000000000	100.0000000000	0.0000000000
High Card	Flush	0.099030	0.0000000000	100.0000000000	0.0000000000
High Card	Full House	0.069397	0.0000000000	100.0000000000	0.0000000000
High Card	4 of a Kind	0.011239	0.0000000000	100.0000000000	0.0000000000
High Card	St. Flush	0.000775	0.0000000000	100.0000000000	0.0000000000
One Pair	High Card	21.083614	100.0000000000	0.0000000000	0.0000000000
One Pair	One Pair	17.910670	49.9979235976	49.9979235976	0.0041528048
One Pair	Two Pair	2.029817	0.0000000000	100.0000000000	0.0000000000
One Pair	3 of a Kind	0.911149	0.0000000000	100.0000000000	0.0000000000
One Pair	Straight	0.165103	0.0000000000	100.0000000000	0.0000000000
One Pair	Flush	0.082681	0.0000000000	100.0000000000	0.0000000000
One Pair	Full House	0.062561	0.0000000000	100.0000000000	0.0000000000
One Pair	4 of a Kind	0.010661	0.0000000000	100.0000000000	0.0000000000
One Pair	St. Flush	0.000647	0.0000000000	100.0000000000	0.0000000000
Two Pair	High Card	2.345956	100.0000000000	0.0000000000	0.0000000000
Two Pair	One Pair	2.029817	100.0000000000	0.0000000000	0.0000000000
Two Pair	Two Pair	0.234280	49.9980157418	49.9980157418	0.0039685164
Two Pair	3 of a Kind	0.107367	0.0000000000	100.0000000000	0.0000000000
Two Pair	Straight	0.018371	0.0000000000	100.0000000000	0.0000000000
Two Pair	Flush	0.009200	0.0000000000	100.0000000000	0.0000000000
Two Pair	Full House	0.007506	0.0000000000	100.0000000000	0.0000000000
Two Pair	4 of a Kind	0.001333	0.0000000000	100.0000000000	0.0000000000
Two Pair	St. Flush	0.000072	0.0000000000	100.0000000000	0.0000000000
3 of a Kind	High Card	1.029252	100.0000000000	0.0000000000	0.0000000000
3 of a Kind	One Pair	0.911149	100.0000000000	0.0000000000	0.0000000000
3 of a Kind	Two Pair	0.107367	100.0000000000	0.0000000000	0.0000000000
3 of a Kind	3 of a Kind	0.048878	50.0000000000	50.0000000000	0.0000000000
3 of a Kind	Straight	0.008060	0.0000000000	100.0000000000	0.0000000000
3 of a Kind	Flush	0.004036	0.0000000000	100.0000000000	0.0000000000
3 of a Kind	Full House	0.003479	0.0000000000	100.0000000000	0.0000000000
3 of a Kind	4 of a Kind	0.000592	0.0000000000	100.0000000000	0.0000000000
3 of a Kind	St. Flush	0.000032	0.0000000000	100.0000000000	0.0000000000
Straight	High Card	0.197921	100.0000000000	0.0000000000	0.0000000000
Straight	One Pair	0.165103	100.0000000000	0.0000000000	0.0000000000
Straight	Two Pair	0.018371	100.0000000000	0.0000000000	0.0000000000
Straight	3 of a Kind	0.008060	100.0000000000	0.0000000000	0.0000000000
Straight	Straight	0.001597	48.0609715367	48.0609715367	3.8780569267
Straight	Flush	0.000775	0.0000000000	100.0000000000	0.0000000000
Straight	Full House	0.000543	0.0000000000	100.0000000000	0.0000000000
Straight	4 of a Kind	0.000088	0.0000000000	100.0000000000	0.0000000000
Straight	St. Flush	0.000006	0.0000000000	100.0000000000	0.0000000000

Flush	High Card	0.099030	100.0000000000	0.0000000000	0.0000000000
Flush	One Pair	0.082681	100.0000000000	0.0000000000	0.0000000000
Flush	Two Pair	0.009200	100.0000000000	0.0000000000	0.0000000000
Flush	3 of a Kind	0.004036	100.0000000000	0.0000000000	0.0000000000
Flush	Straight	0.000775	100.0000000000	0.0000000000	0.0000000000
Flush	Flush	0.000498	49.9614056912	49.9614056912	0.0771886177
Flush	Full House	0.000272	0.0000000000	100.0000000000	0.0000000000
Flush	4 of a Kind	0.000044	0.0000000000	100.0000000000	0.0000000000
Flush	St. Flush	0.000004	0.0000000000	100.0000000000	0.0000000000
Full House	High Card	0.069397	100.0000000000	0.0000000000	0.0000000000
Full House	One Pair	0.062561	100.0000000000	0.0000000000	0.0000000000
Full House	Two Pair	0.007506	100.0000000000	0.0000000000	0.0000000000
Full House	3 of a Kind	0.003479	100.0000000000	0.0000000000	0.0000000000
Full House	Straight	0.000543	100.0000000000	0.0000000000	0.0000000000
Full House	Flush	0.000272	100.0000000000	0.0000000000	0.0000000000
Full House	Full House	0.000252	50.0000000000	50.0000000000	0.0000000000
Full House	4 of a Kind	0.000044	0.0000000000	100.0000000000	0.0000000000
Full House	St. Flush	0.000002	0.0000000000	100.0000000000	0.0000000000
4 of a Kind	High Card	0.011239	100.0000000000	0.0000000000	0.0000000000
4 of a Kind	One Pair	0.010661	100.0000000000	0.0000000000	0.0000000000
4 of a Kind	Two Pair	0.001333	100.0000000000	0.0000000000	0.0000000000
4 of a Kind	3 of a Kind	0.000592	100.0000000000	0.0000000000	0.0000000000
4 of a Kind	Straight	0.000088	100.0000000000	0.0000000000	0.0000000000
4 of a Kind	Flush	0.000044	100.0000000000	0.0000000000	0.0000000000
4 of a Kind	Full House	0.000044	100.0000000000	0.0000000000	0.0000000000
4 of a Kind	4 of a Kind	0.000007	50.0000000000	50.0000000000	0.0000000000
4 of a Kind	St. Flush	0.000000	0.0000000000	100.0000000000	0.0000000000
St. Flush	High Card	0.000775	100.0000000000	0.0000000000	0.0000000000
St. Flush	One Pair	0.000647	100.0000000000	0.0000000000	0.0000000000
St. Flush	Two Pair	0.000072	100.0000000000	0.0000000000	0.0000000000
St. Flush	3 of a Kind	0.000032	100.0000000000	0.0000000000	0.0000000000
St. Flush	Straight	0.000006	100.0000000000	0.0000000000	0.0000000000
St. Flush	Flush	0.000004	100.0000000000	0.0000000000	0.0000000000
St. Flush	Full House	0.000002	100.0000000000	0.0000000000	0.0000000000
St. Flush	4 of a Kind	0.000000	100.0000000000	0.0000000000	0.0000000000
St. Flush	St. Flush	0.000000	45.4268292683	45.4268292683	9.1463414634

Summary of Player Hand Performance at Showdown

Player Hand	Frequency	Win %	Loss %	Tie %
High Card	50.117739	25.2132750287	74.7709447595	0.0157802118
One Pair	42.256903	71.0856087558	28.9126310699	0.0017601743
Two Pair	4.753902	94.5099212035	5.4898832216	0.0001955749
3 of a Kind	2.112845	98.0765946968	1.9234053032	0.0000000000
Straight	0.392465	99.4286205031	0.5555992851	0.0157802118
Flush	0.196540	99.7103399613	0.2894644638	0.0001955749
Full House	0.144058	99.8801978235	0.1198021765	0.0000000000
4 of a Kind	0.024010	99.9831446177	0.0168553823	0.0000000000
St. Flush	0.001539	99.9988330696	0.0009713554	0.0001955749
Total	100.000000	49.9956379514[**]	49.9956379514[††]	0.0087240972[‡‡]

[**] Sum of Frequency * Win %

[†] Sum of Frequency * Loss %
[‡] Sum of Frequency * Tie %